Sunday, December 15, 2024 7:38 PM

$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ → $A\vec{v} = \lambda\vec{v}$

$\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}\boxed{\begin{pmatrix} 1 \\ 2 \end{pmatrix}} = \boxed{5}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\begin{pmatrix} 1 + 4 \\ 4 + 6 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$

$\begin{pmatrix} 5 \\ 10 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$

$A\vec{v} = \lambda\vec{v}$

$A\vec{v} - \lambda\vec{v} = 0$

$(A - \lambda)\vec{v} = 0$

$\vec{v}\,(A - \lambda) = 0$

Matriks $\boxed{\text{Skalar}}$ → Matriks

$\boxed{(A - \lambda I)\vec{v} = 0}$

$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$

$\begin{pmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{pmatrix} \to (1-\lambda)(3-\lambda) - 8$

$= 3 - \lambda - 3\lambda + \lambda^2 - 8$

$0 = -5 - 4\lambda + \lambda^2$

$\boxed{\det(A - \lambda I) = 0}$

$\lambda^2 - 4\lambda - 5 = 0$

$(\lambda - 5)(\lambda + 1) = 0$

$\lambda_1 = 5 \vee \lambda_2 = -1$

# $\lambda = 5$

Nilai Eigen $= \{5 ; -1\}$

$\begin{pmatrix} 1-5 & 2 \\ 4 & 3-5 \end{pmatrix}$

$A - \lambda I$

$\boxed{\begin{pmatrix} -4 & 2 \\ 4 & -2 \end{pmatrix}} \underset{H_1(\frac{1}{2})}{\overset{H_{21(1)}}{\sim}} \boxed{\begin{pmatrix} -2 & 1 \\ 0 & 0 \end{pmatrix}}$ $-2x_1 + x_2 = 0$ $\boxed{x_2 = 2x_1}$

$(A - \lambda I)\vec{v} = \vec{0}$

$\begin{pmatrix} -4 & 2 \\ 4 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$4x_1 - 2x_2 = 0$

$\frac{4x_1}{2} = \frac{2x_2}{2}$

$\boxed{2x_1 = x_2}$

misal $x_1 = a$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a \\ 2a \end{pmatrix}$

$= a\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

# SPD..?

Nilai Eigen $= \{5 ; -1\}$

Vektor Eigen $= \left\{\begin{pmatrix} 1 \\ 2 \end{pmatrix}; \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\}$

$\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} = M_{n \times n}$

$= M_{2 \times 2}$

$-\frac{1}{3}\begin{pmatrix} -1 & -1 \\ -2 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$

$MD = \boxed{P^{-1}AP}$

$= \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$

$\frac{1}{\det}\,\text{adj}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

↳ adj $= -\frac{1}{3}\begin{pmatrix} -1 & -1 \\ -2 & 1 \end{pmatrix}$

$= \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{pmatrix}$

# $\lambda = -1$

$\begin{pmatrix} 2 & 2 \\ 4 & 4 \end{pmatrix} \underset{H_1(\frac{1}{2})}{\overset{H_{21(-2)}}{\sim}} \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

$2x_1 + 2x_2 = 0$

$\frac{2x_1}{2} = -\frac{2x_2}{2}$

$x_1 = -x_2$

misal $x_1 = a$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a \\ -a \end{pmatrix}$

$= a\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Vektor Eigen $= \left\{\begin{pmatrix} 1 \\ 2 \end{pmatrix}; \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\}$

$\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = -1\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\begin{pmatrix} 1 - 2 \\ 4 - 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} \frac{1}{3} + \frac{4}{3} & \frac{2}{3} + 1 \\ \frac{2}{3} - \frac{4}{3} & \frac{4}{3} - 1 \end{pmatrix}$

$= \begin{pmatrix} \frac{5}{3} & \frac{5}{3} \\ -\frac{2}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$

$= \begin{pmatrix} \frac{5}{3} + \frac{10}{3} & \frac{5}{3} - \frac{5}{3} \\ -\frac{2}{3} + \frac{2}{3} & -\frac{2}{3} - \frac{1}{3} \end{pmatrix}$

$= \begin{pmatrix} \frac{15}{3} & 0 \\ 0 & -\frac{3}{3} \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix}$

$x_1' = -5x_1 + x_2$

$x_2' = 4x_1 - 2x_2$ → $\overset{A}{\boxed{\begin{pmatrix} -5 & 1 \\ 4 & -2 \end{pmatrix}}}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$(A - \lambda I)\vec{v} = 0$

$\begin{pmatrix} -5-\lambda & 1 \\ 4 & -2-\lambda \end{pmatrix}$

$(-5-\lambda)(-2-\lambda) - (1)(4)$

$10 + 5\lambda + 2\lambda + \lambda^2 - 4$

$\lambda^2 + 7\lambda + 6 = 0$

$(\lambda + 6)(\lambda + 1) = 0$

$\lambda_1 = -6 \vee \lambda_2 = -1$

$\lambda = -6$

$\begin{pmatrix} -5+6 & 1 \\ 4 & -2+6 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ 4 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$x_1 + x_2 = 0$

$x_1 = -x_2$

$\lambda = -1$

$\begin{pmatrix} -5+1 & 1 \\ 4 & -2+1 \end{pmatrix}$

$\begin{pmatrix} -4 & 1 \\ 4 & -1 \end{pmatrix}$

$4x_1 - x_2 = 0$

$4x_1 = x_2$

Materi 14 : Eigen $\boxed{A\vec{v} = \lambda\vec{v}}$

$(A - \lambda I)\vec{v} = 0$

1. cari $A - \lambda I$
2. cari determinan $A - \lambda I$
   ↳ eigenvalue
3. untuk setiap eigenvalue, masukkan ke $A - \lambda I$
   ↳ eigenvector

Diagonalisasi

misal $M_{n \times n}$,

maka dia bisa didiagonalisasikan jika dan hanya jika ada $n$ vektor eigen yang bebas linier.

$$\boxed{D = P^{-1} A P}$$ → gabungan vektor eigen

Materi 15: SPDOS

$f'(x) = A \cdot f(x)$

$f(x) = C e^{\lambda x}$

$C$ ↓ vektor eigen; $\lambda$ ↘ nilai eigen

misal $f'(x) = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} f(x)$

untuk setiap pasang eigenvector dan eigenvalue, → $F(x) = A C_1 e^{\lambda_1 x} + B C_2 e^{\lambda_2 x}$

eval = {8, -1}

evec = $\left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -5/2 \\ 1 \end{pmatrix} \right\}$

$f(x) = A \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{8x} + B \begin{pmatrix} -5/2 \\ 1 \end{pmatrix} e^{-x}$

$A C_1 + B C_2 = f(0)$

$x'_1 = x_1 + 2x_2 \quad x_1(0) = 0$
$x'_2 = 3x_1 + 2x_2 \quad x_2(0) = -4$

$\vec{x}' = \underbrace{\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}}_{\text{EVAL + EVEC}} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad x(0) = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$

$A - \lambda I$

$= \begin{pmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{pmatrix}$

$(1-\lambda)(2-\lambda) - 6$

$2 - \lambda - 2\lambda + \lambda^2 - 6$

$\lambda^2 - 3\lambda - 4 = 0$

$(\lambda - 4)(\lambda + 1)$

$\lambda_1 = 4 \quad \lambda_2 = -1$

# $\lambda = -1$

$\begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}$

$2x_1 + 2x_2 = 0$

$2x_1 = -2x_2$

$x_1 = -x_2$

misal $x_2 = p$

$\begin{pmatrix} -p \\ p \end{pmatrix} = p \begin{pmatrix} -1 \\ 1 \end{pmatrix}$,

# $\lambda = 4$

$\begin{pmatrix} -3 & 2 \\ 3 & -2 \end{pmatrix}$

$3x_1 - 2x_2 = 0$

$3x_1 = 2x_2$

$x_1 = \frac{2}{3} x_2$

misal $x_2 = p$

$\begin{pmatrix} \frac{2}{3}p \\ p \end{pmatrix} = p \begin{pmatrix} \frac{2}{3} \\ 1 \end{pmatrix}$ //

$\underbrace{\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}}_{A} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = -1 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ → eigenvector; $-1$ ↑ eigenvalue

$\begin{pmatrix} 1-2 \\ 3-2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$x_1 + x_2 = 0$

$x_1 = -x_2$

misal $x_2 = p$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -p \\ p \end{pmatrix} \longleftrightarrow p \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$4x_1 - x_2 = 0$

$4x_1 = x_2$

misal $x_1 = p$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} p \\ 4p \end{pmatrix} \longleftrightarrow p \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$\text{EVAL} = \{-6 ; -1\}$

$\text{EVEC} = \left\{ \begin{pmatrix} -1 \\ 1 \end{pmatrix} ; \begin{pmatrix} 1 \\ 4 \end{pmatrix} \right\}$

$k C e^{\lambda x}$ — eigenvalue ($\lambda$), eigen vector ($C$)

$\vec{x}' = \begin{pmatrix} -5 & 1 \\ 4 & -2 \end{pmatrix} \vec{x}, \quad \vec{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\vec{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\vec{x} = A \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-6x} + B \begin{pmatrix} 1 \\ 4 \end{pmatrix} e^{-x} \longrightarrow \vec{x} = -\frac{2}{5} \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-6x} + \frac{3}{5} \begin{pmatrix} 1 \\ 4 \end{pmatrix} e^{-x}$

$\vec{x} = \begin{pmatrix} 2/5 \\ -2/5 \end{pmatrix} e^{-6x} + \begin{pmatrix} 3/5 \\ 12/5 \end{pmatrix} e$

$\begin{pmatrix} -1 \\ 1 \end{pmatrix} A + \begin{pmatrix} 1 \\ 4 \end{pmatrix} B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\begin{pmatrix} -A + B \\ A + 4B \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$-A + B = 1$

$-A = 1 - B$

$A = B - 1$

$A + 4B = 2$

$B - 1 + 4B = 2$

$5B = 3$

$B = \frac{3}{5}$

$-A + \frac{3}{5} = 1$

$-A = 1 - \frac{3}{5}$

$-1 \times \; -A = \frac{2}{5} \times -1$

$A = -\frac{2}{5}$

Diberikan sistem persamaan diferensial linier orde satu sebagai berikut

$$\begin{aligned} f_1'(x) &= 3f_1(x) + 10f_2(x) \\ f_2'(x) &= 2f_1(x) + 4f_2(x) \end{aligned} \quad \text{dimana} \quad f_1(0) = 0 \quad \text{dan} \quad f_2(0) = 1$$

Tentukan nilai $f_1(x)$ dan $f_2(x)$

$f'(x) = \begin{pmatrix} 3 & 10 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix}$ $\quad f_1(0) = 0$, $f_2(0) = 1$ $\quad f(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$(A - \lambda I)\vec{v} = 0$

$\begin{pmatrix} 3 - \lambda & 10 \\ 2 & 4 - \lambda \end{pmatrix}$

$(3 - \lambda)(4 - \lambda) - 20$

$12 - 3\lambda - 4\lambda + \lambda^2 - 20$

$\lambda^2 - 7\lambda - 8 = 0$

$(\lambda - 8)(\lambda + 1) = 0$

$\lambda_1 = 8 \vee \lambda_2 = -1$

# $\lambda = 8$

$\begin{pmatrix} 3 - 8 & 10 \\ 2 & 4 - 8 \end{pmatrix} \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} -5 & 10 \\ 2 & -4 \end{pmatrix} \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$2f_1(x) - 4f_2(x) = 0$

# $\lambda = -1$

$\begin{pmatrix} 3 + 1 & 10 \\ 2 & 4 + 1 \end{pmatrix} \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 4 & 10 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$2f_1(x) = -5f_2(x)$

$$e^{-x}$$

$x_2 = p$

$$\begin{pmatrix} -p \\ p \end{pmatrix} = p\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$A \quad \begin{pmatrix} 1-2 \\ 3-2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = -1\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -1+2 \\ -3+2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

EVAL $= \{4, -1\}$

EVEC $= \left\{\begin{pmatrix} 2/3 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}\right\}$

$$F(x) = A\begin{pmatrix} 2/3 \\ 1 \end{pmatrix}e^{4x} + B\begin{pmatrix} -1 \\ 1 \end{pmatrix}e^{-x} \rightarrow -\frac{12}{5}\begin{pmatrix} 2/3 \\ 1 \end{pmatrix}e^{4x} - \frac{8}{5}\begin{pmatrix} -1 \\ 1 \end{pmatrix}e^{-x}$$

$$F(0) = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \qquad A = -\frac{12}{5} \qquad B = -\frac{8}{5}$$

$$\begin{pmatrix} -8/5 \\ -12/5 \end{pmatrix}e^{4x} - \begin{pmatrix} -8/5 \\ 8/5 \end{pmatrix}e^{-x}$$

$$A\begin{pmatrix} 2/3 \\ 1 \end{pmatrix} + B\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \qquad -\frac{\cancel{12}^{4}}{5} \times \frac{2}{\cancel{3}} = -\frac{8}{5}$$

$$\tfrac{2}{3}A - B = 0$$

$$\tfrac{2}{3}A = B$$

$$A = \frac{3}{2}B$$

$$A + B = -4 \qquad A - \frac{8}{5} = -4$$

$$\frac{3}{2}B + B = -4 \qquad A = -4 + \frac{8}{5}$$

$$\frac{5}{2}B = -4 \qquad = -\frac{20}{5} + \frac{8}{5}$$

$$B = -\frac{8}{5} \qquad = -\frac{12}{5}$$

$\lambda^2 - 7\lambda - 8 = 0$

$(\lambda - 8)(\lambda + 1) = 0$

$\lambda_1 = 8 \vee \lambda_2 = -1$

$2f_1(x) - 4f_2(x) = 0$

$2f_1(x) = 4f_2(x)$

$f_1(x) = 2f_2(x)$

misal $f_2(x) = p$

$\begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} = \begin{pmatrix} 2p \\ p \end{pmatrix} = p \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ //

$2f_1(x) = -5f_2(x)$

$f_1(x) = \frac{-5f_2(x)}{2}$

misal $f_2(x) = p$

$\begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} = \begin{pmatrix} -\frac{5}{2}p \\ p \end{pmatrix} = p \begin{pmatrix} -5/2 \\ 1 \end{pmatrix}$ //

eval $= \{8, -1\}$

evec $= \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -5/2 \\ 1 \end{pmatrix} \right\}$

$f(x) = A \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{8x} + B \begin{pmatrix} -5/2 \\ 1 \end{pmatrix} e^{-x} \longleftrightarrow f(x) = \frac{5}{9} \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{8x} + \frac{4}{9} \begin{pmatrix} -5/2 \\ 1 \end{pmatrix} e^{-x}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = A \begin{pmatrix} 2 \\ 1 \end{pmatrix} + B \begin{pmatrix} -5/2 \\ 1 \end{pmatrix}$

$f(x) = \begin{pmatrix} 10/9 \\ 5/9 \end{pmatrix} e^{8x} + \begin{pmatrix} -10/9 \\ 4/9 \end{pmatrix} e^{-x}$

$2A - \frac{5}{2}B = 0 \longleftrightarrow 2A = \frac{5}{2}B$

$A = \frac{5}{4}B$

$A + B = 1$

$\frac{5}{4}B + B = 1$

$\frac{9}{4}B = 1$

$B = \frac{4}{9}$ ✓

$A + \frac{4}{9} = 1$

$A = 1 - \frac{4}{9}$

$A = \frac{5}{9}$ ✓